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RESUMMATION FOR DRELL-YAN DIFFERENTIAL DISTRIBUTIONS [1]

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ABSTRACT

We address the question of resummation for Drell-Yan differential distributions, specifically the case of zero rapidity or x_F. We find that we can use the inclusive hard part in order to resum the large corrections due to soft collinear gluon emission. This greatly simplifies resummation efforts for these distributions. The one loop rapidity distribution is investigated to determine the quality of our approximation, which is found to be very good at zero rapidity.

The resummation of large corrections that occur near the boundary of phase space in Drell-Yan has been a topic of investigation for quite a few years. In particular the case of the inclusive cross section has been thoroughly investigated. Here we address the question of resummation for differential distributions, which allow a wider comparison with experiment. We find that for the case of the rapidity distribution at zero rapidity, the resummed hard part of the inclusive case can be used.

Let us consider a differential Drell-Yan cross section, in terms of Q^2 and some dimensionless variable $\eta(q,s)$, which we assume to be a smooth function of the virtual photon momentum q^μ at all values of q^μ that are allowed by the requirement $q^2 = Q^2$. Examples are $x_F = 2q_z/\sqrt{s}$ and the rapidity, $Y = (1/2)\ln[(q_0+q_z)/(q_0-q_z)]$. The arguments we present in the following are phrased generally, but for our present purposes one should read $\bar{\eta} = Y = 0$, or $\bar{\eta} = x_F = 0$. The factorization theorem [1] gives for this differential cross section the general form

$$\frac{d\sigma}{dQ^2 d\eta} = \sigma_0 \int_0^1 \frac{dx_1}{x_1}\frac{dx_2}{x_2} f_{a/A}(x_1) f_{b/B}(x_2) H(x_1p_1, x_2p_2, Q^2, \bar{\eta}). \tag{1}$$

Here we have suppressed the scale dependence in the parton distributions $f(x)$. σ_0 may be taken as the Born cross section, so that the hard-scattering function $H(x_1p_1, x_2p_2, Q^2, r)$ may be taken to be dimensionless. H is a sum over partonic matrix elements, subtracted to eliminate divergences associated with the incoming hadrons. As such, it may always be put in the form of a sum over terms, each of

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which contains an integral over a (possibly trivial) hadronic final state, whose total momentum is $x_1p_1 + x_2p_2 - q$, with q^μ the momentum of the virtual photon. Thus, schematically, we have

$$\begin{aligned} H(x_1p_1, x_2p_2, Q^2, \bar{\eta}) &= \sum_i \int \frac{\mathrm{d^4k}}{(2\pi)^4} |M(x_1p_1, x_2p_2, k)|_i^2 \\ &\times \delta(Q^2 - (x_1p_1 + x_2p_2 - k)^2)\delta(\bar{\eta} - \eta(x_1p_1 + x_2p_2 - k, s)). \end{aligned} \quad (2)$$

Subscript i labels contributions to all final states which include the virtual photon. In each case, we integrate over the final state, subject only to fixed total momentum k for the hadrons. The difference between the differential cross section in η and the inclusive cross section $\mathrm{d}\sigma/\mathrm{dQ}^2$ is entirely contained in the second delta function. Without it, the integral and sum over the $|H|_i^2$ reproduce the inclusive hard part,

$$\sum_i \int \frac{\mathrm{d^4k}}{(2\pi)^4} |M(x_1p_1, x_2p_2, k)|_i^2 \delta(Q^2 - (x_1p_1 + x_2p_2 - k)^2) = \omega(z), \quad (3)$$

where $z = Q^2/x_1x_2s$. Although the integral over k^μ is in general singular for a given final state, the factorization theorem assures that the sum over final states produces a distribution in z that is infrared finite.

Now we note that as $z \to 1$, the phase space allowed for the k^μ integral vanishes. It is easy to see that for solutions to the photon mass shell condition $0 = Q^2 - (x_1p_1 + x_2p_2 - k)$ with $k_0 > 0$ all k^μ-components vanish as $1 - z$. Given our assumption that $\eta(q)$ is a smooth function of q^μ in the allowed range, we can expand $\eta(x_1p_1 + x_2p_2 - k)$ and the delta function in η about $k = 0$,

$$\begin{aligned} \delta(\bar{\eta} - \eta(x_1p_1 + x_2p_2 - k, s)) &= \delta(\bar{\eta} - \eta(x_1p_1 + x_2p_2)) \\ &+ k^\mu \frac{\partial \eta(q)}{\partial q^\mu}|_{q = x_1p_1 + x_2p_2} \delta'(\bar{\eta} - \eta(x_1p_1 + x_2p_2)) + \ldots, \end{aligned} \quad (4)$$

Substituting this expansion into the factorization formula, we keep only the first term. Dimensional analysis shows that the second, and succeeding terms in the expansion are supressed by factors of $1 - z$, due to their extra factors of k^μ in their respective phase space integrals. Such terms are not singular as $z \to 1$, and need not be included in the resummation formula. Our result, then, keeping all terms that are singular as $z \to 1$, is

$$\begin{aligned} \left(\frac{\mathrm{d}\sigma}{\mathrm{dQ^2 d}\eta}\right)^{(\mathrm{resum})} &= \sigma_0 \int_0^1 \frac{\mathrm{dx_1}}{\mathrm{x_1}} \frac{\mathrm{dx_2}}{\mathrm{x_2}} f_{a/A}(x_1) f_{b/B}(x_2) \\ &\times \delta(\bar{\eta} - \eta(x_1p_1 + x_2p_2)\omega(z)^{(\mathrm{resum})}. \end{aligned} \quad (5)$$

This is the result we set out to prove. In this expression, one can simply take the hard part $\omega(z)^{(\mathrm{resum})}$ which is available in the literature [2, 3].

In order to get a better idea of how well the approximation works, we investigate the case of the one-loop Drell-Yan rapidity distribution, at zero rapidity, in detail. We restrict ourselves to the $q\bar{q}$ channel. The exact answer was calculated first in [4], and cast in a simpler form in [5]. For the approximate expression we use

Eq. (5), where $\bar{\eta}$ should be taken as the rapidity and set to zero. The quantity we plot is the one loop K-factor, defined by

$$\left(\frac{d\sigma^{(0)}}{dQ^2dY}|_{Y=0}\right) K^{(1)}(\tau, Y=0) = \frac{d\sigma^{(1)}}{dQ^2dY}|_{Y=0}, \tag{6}$$

where $\tau = Q^2/s$. For simplicity we chose the Q^2-independent parton densities of Khalafi and Stirling [6], together with a one-loop α_S with four flavours and $\Lambda = 0.5$ GeV. We plot the one-loop K-factor in Fig.1 for $p\bar{p}$ scattering at $\sqrt{s} = 630$ GeV.

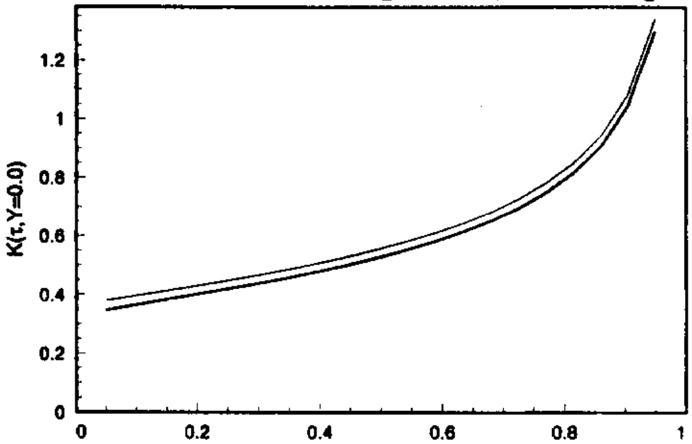

Fig.1. Approximate vs. exact one-loop K-factor at zero rapidity, $p\bar{p}$ scattering, at $\sqrt{s} = 630$ GeV. The top line is the exact result, the bottom one our approximation.

From Fig.1 it is clear that the approximation works remarkably well over the whole range in τ. We have also investigated the same K-factor in pp scattering at fixed target energies ($\sqrt{s} \approx 30$ GeV), and find similarly good agreement. Thus we feel confident in presenting Eq. (5) as a good approximation to the resummed DY rapidity distribution at zero rapidity (or x_F).

We should add however that when the rapidity of the vector boson is significantly different from zero we find that the approximation works less well. This case will be discussed in a forthcoming paper [7].

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